本考科禁用掌上型計算機

1. Find each integral.

(a) (5%) $\int \frac{4}{x^2+9} dx$ 　 (b) (5%) $\int \frac{4x}{x^2+9} dx$ 　 (c)(5%) $\int \frac{1}{1+e^x} dx$.

2. (10%) Find $\lim\limits_{x \to 0^+} (\sin x)^x$.

3. (10%) Find the tangent line to the graph given by $x^2(x^2 + y^2) = y^2$ at the point $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

4. (10%) If $f(x) = \int_2^x \frac{dt}{\sqrt{1+t^4}}$, find $(f^{-1})'(0)$.

5. (15%) Let R be the region bounded by the square with vertices $(0, 1), (1, 2), (2, 1)$, and $(1, 0)$. Evaluate the integral

$$\iint_R (x+y)^2 \sin^2(x-y) dA.$$

6. (15%) Evaluate $\int_1^\infty (1-x)e^{-x} dx$.

7. Let

$$f(x, y) = \begin{cases} \frac{5x^2y}{x^3+y^3} & , (x,y) \neq (0,0) \\ 0 & , (x,y) = (0,0) \end{cases}$$

Prove that
(a) (5%) $f_x(0, 0)$ exists, and
(b) (5%) f is not differentiable at $(0,0)$.

8. (15%) Find the extreme values of

$$f(x, y) = x^2 + 2y^2 - 2x + 3$$

subject to the constraint $x^2 + y^2 \leq 10$.