Formulation of quantum phase on the IBM-quantum: qubit number versus phase-estimation accuracy

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Quantum Phase Estimation (QPE) [1] is a fundamental algorithm in quantum computing, playing a pivotal role in a wide range of quantum applications [2,3]. In this work, employing IBM-Quantum simulator, we examine the evolution of electrons generated by non-Abelian [4] quantum phase generator traveling along the paths of quantum square ring (QSR) [5]. These paths include two-legs (L-shaped), three-legs (Π-shaped) and cyclic systems [see Fig. (a)]. We propose a quantum computing circuit that combines the rotation gate (RX and RY gate) and QPE structure [see Fig. (b)] for measuring the quantum phase, which can be realized by electron spin evolution [6] in the QSR. This circuit enables the *phase-to-amplitude conversion*. To illustrate the conversion, we introduce two phase parameters δ and η that can be read out from the output probabilities of the binary states [see Fig. (c)]. The circuit facilitates formulating and determining the total phase of the desired paths. We also analyze the accuracy of our circuit. It is found that the accuracy of phase measurement depends on the number of assigned and adopted qubits. Specifically, the more assigned qubits, the less errors get from the circuit. However, high accuracy can be achieved only by adopting the first few positions of qubits. In some cases, adopting all the assigned qubits will result in slight phase-error deviations [see Fig. (d)]. In other words, other unadopted qubits are used for auxiliary purposes. The proposed method and the investigated model of the QSR can be extended to detect quantum entanglement, which is an extension of the two-electron case from this study. This work is supported by National Science and Technology Council under Grant No. 110-2112-M-034-001-MY3.

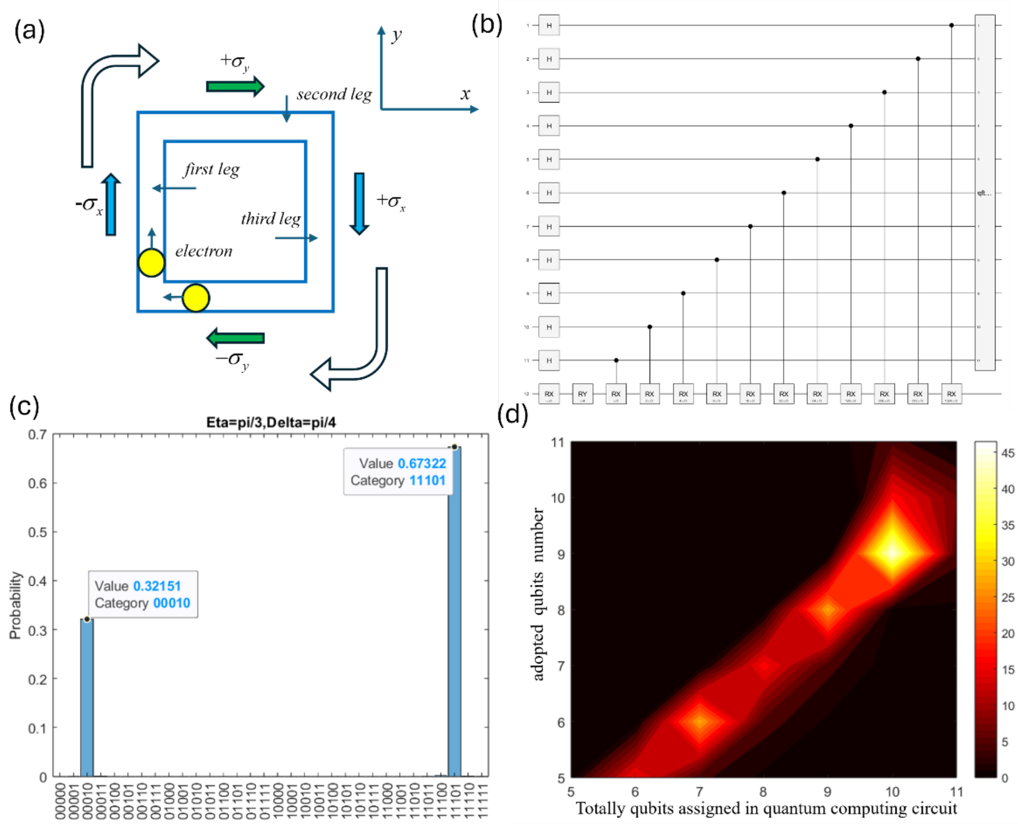

Figure: (a) The electron travels clockwise along a single path in the quantum square ring. (b) The structure of quantum computing circuit combined with quantum phase estimation to simulate three-leg path. (c) The amplitude also the probability corresponds to two respectively binary state with phase parameter $\eta=\pi/3,\ \delta=\pi/4$. (d) The color in the image represents the percentage error, showing the relationship between the total number of qubits assigned in the quantum computing circuit and the adopted qubits.

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