Quantum squeezing and entanglement in coherent atomic systems

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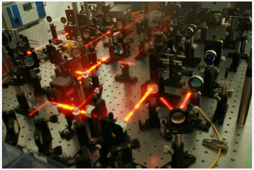
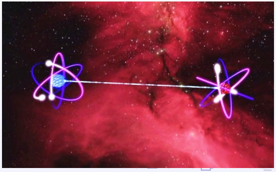
14, May, 2018
Physics department, National Dong Hua university





Acknowledgement

Ite A. Yu: Department of Physics, National Tsing Hua university, Hsinchu 300, Taiwan.
Ray-Kuang Lee: Institute of photonics technologies, National Tsing Hua university, Hsinchu 300, Taiwan.
Financial supports from **National Center for Theoretical Sciences**

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Contents

Electromagnetically Induced Transparency (EIT)

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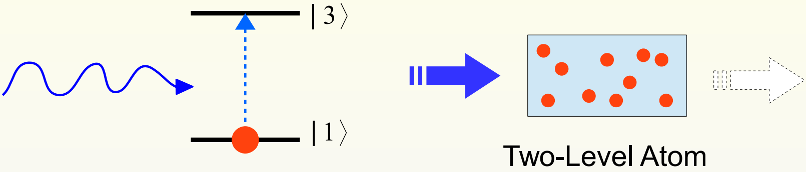

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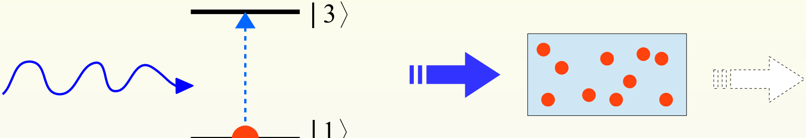
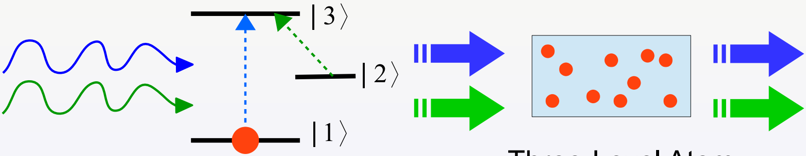


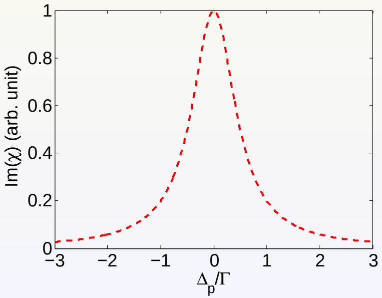


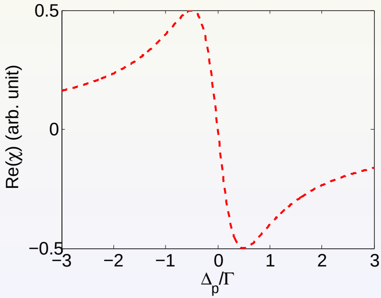
dispersion and absorption

The susceptibility of EIT medium is

$$\chi = \frac{N|\mu|^2}{\epsilon_0 \hbar} \frac{i\left(\gamma - i(\Delta_p - \Delta_c)\right)}{(\Gamma/2 - i\Delta_p)(\gamma - i(\Delta_p - \Delta_c)) + |\Omega_c|^2/4},$$

$$\text{group velocity} : v_g = c\left[n + \frac{\omega_p}{2}\frac{\partial \mathrm{Re}(\chi)}{\partial \omega_p}\right]^{-1}.$$





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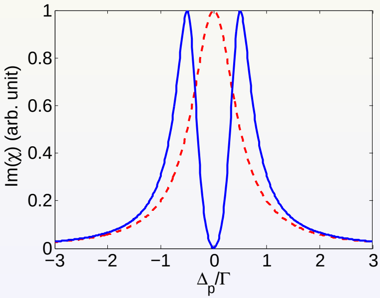
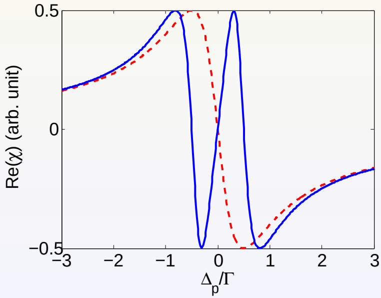




Properties and applications of EIT

Electromagnetically induced transparency is a technique for eliminating the effect of a medium on a propagating beam of electromagnetic radiation.

Optical properties:

- high transmission
- large refractive index
- enhancement of $\chi^{(3)}$ nonlinearity

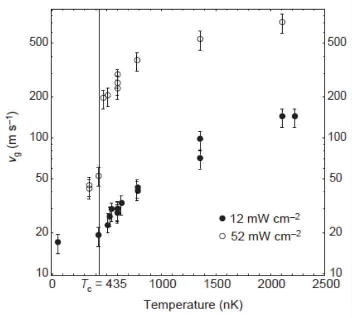
Applications:

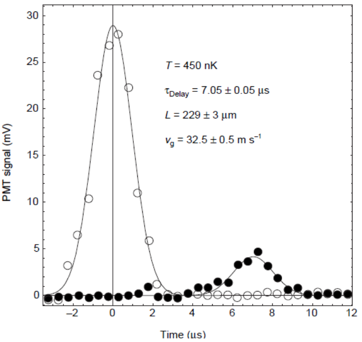
- ultraslow light
- light storage
- quantum memory
- laser without inversion
- precision spectroscopy

Ultraslow light in EIT

Light speed reduction to 17 metres per second in an ultracold atomic gas

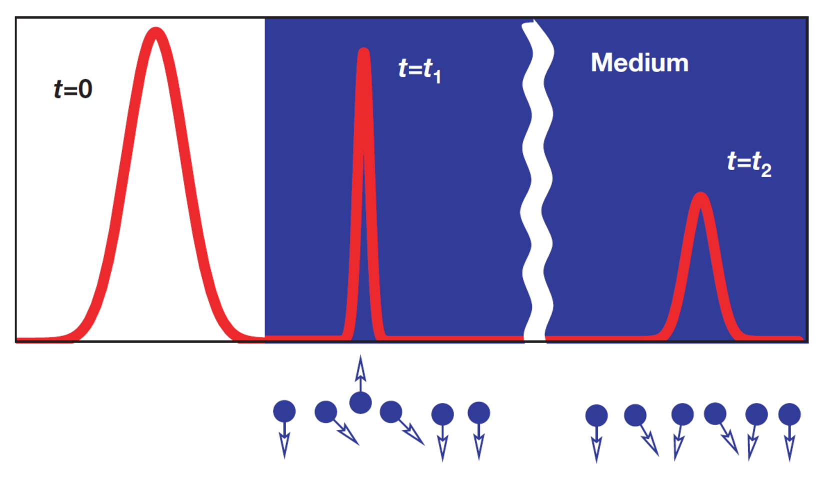
Lene Vestergaard Hau*†, S. E. Harris‡, Zachary Dutton*† & Cyrus H. Behroozi*§





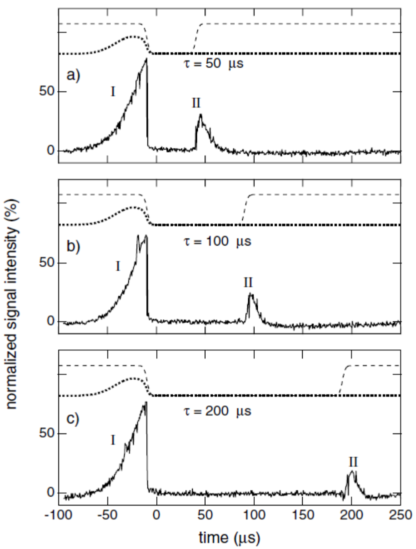
NATURE | VOL 397 | 18 FEBRUARY 1999 | www.nature.com

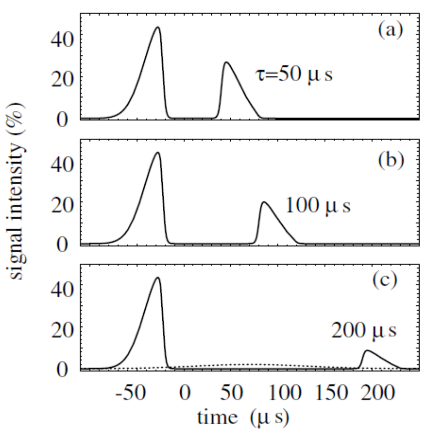
Spatial compression of optical pulse in EIT



Controlling photons using electromagnetically induced transparency, Nature **413**, 273 - 276 (2001).

Storage and retrieval of optical pulse



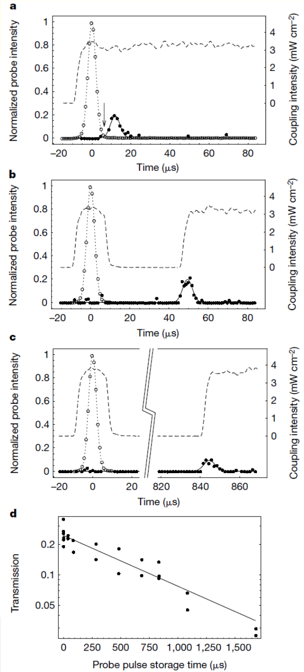


D. F. Phillips et al., PRL 86, 783 (2001)

Observation of coherent optical information storage in an atomic medium using halted light pulses

Chien Liu*†, Zachary Dutton*‡, Cyrus H. Behroozi*† & Lene Vestergaard Hau*†‡

** Rowland Institute for Science, 100 Edwin H. Land Boulevard, Cambridge, Massachusetts 02142, USA*
† Division of Engineering and Applied Sciences, ‡ Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA



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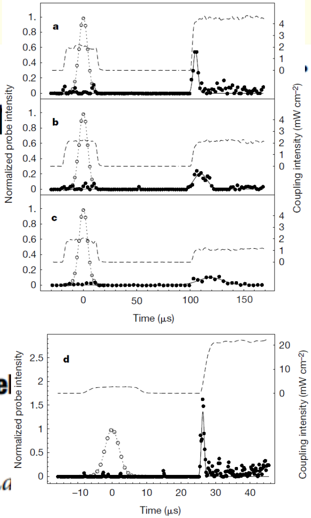

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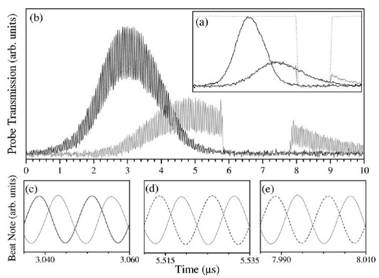
d Applied Sciences, ‡ Department of Physics,
dge, Massachusetts 02138, USA

Preservation of optical phase

PHYSICAL REVIEW A **72**, 033812 (2005)

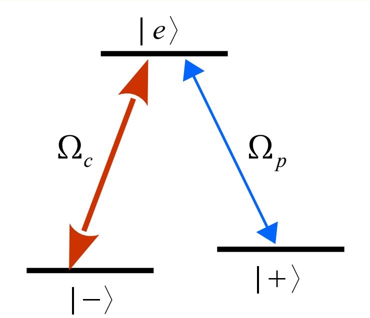
Beat-note interferometer for direct phase measurement of photonic information

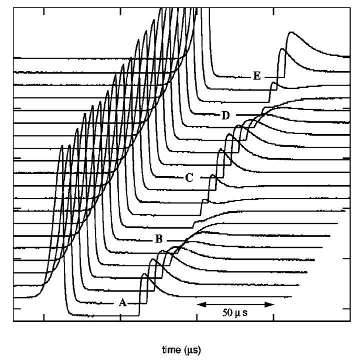
Yong-Fan Chen, Yu-Chen Liu, Zen-Hsiang Tsai, Shih-Hao Wang, and Ite A. Yu*
Department of Physics, National Tsing Hua University, Hsinchu, Taiwan 300, Republic of China
(Received 19 February 2005; published 14 September 2005)



The storage and retrieval of the probe pulse described above is a coherent process [16,17]. This is demonstrated intuitively in Fig. 2(b) with the beat-note interferometer. The black and gray beat notes are the signals from PD1 and PD2, respectively. If the phases of the incoming and outgoing waves of the storage are uncorrelated, there is no beat signal in the retrieved pulse. The phases in different parts of the probe pulse were quantitatively examined, as shown in Figs. 2(c)–2(e). We triggered the oscilloscope by the reference beat note. A rf switch (Mini-Circuits ZFSWHA-1-20) was employed to select the Gaussian peak, and its output provided the trigger signal. The oscilloscope waited a certain delay time and then acquired data from the two detectors. We found the instability of a 1-ms delay of the oscilloscope to be equivalent to a phase jitter of $\pm 1.5°$ at the beat frequency of 80 MHz. The driving frequency of the AOM or the beat frequency is sufficiently stable that we are able to extrapolate the reference beat note in Figs. 2(d) and 2(e). Within the measurement accuracy, the phase of the retrieved wave perfectly evolves from the incoming wave and there is no observable phase jump caused by the switching process.

Phase coherence and control

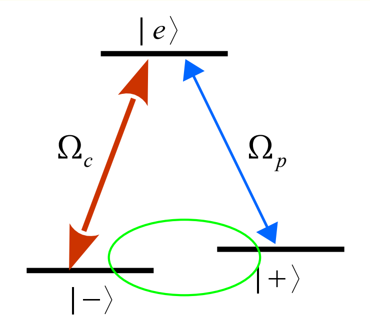


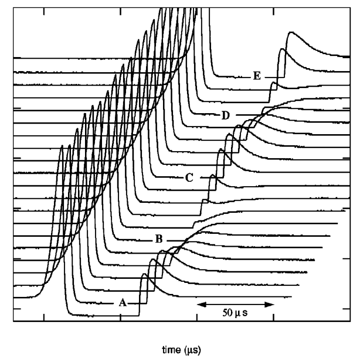


phase shift : $\Phi = (g_+ - g_-)\frac{\mu_B}{\hbar}\int_0^T dt' B(t')$

M.D. Lukin *et al*, Phys. Rev. A **65**, 031802(R) (2002).

Phase coherence and control





$$\text{phase shift}: \Phi = (g_+ - g_-)\frac{\mu_B}{\hbar}\int_0^T dt' B(t')$$

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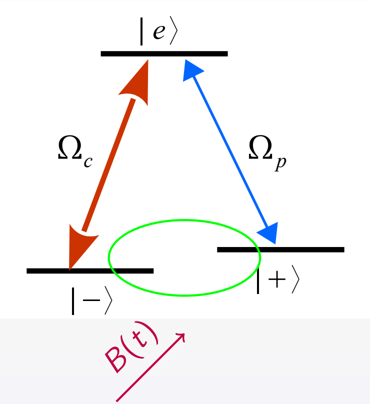

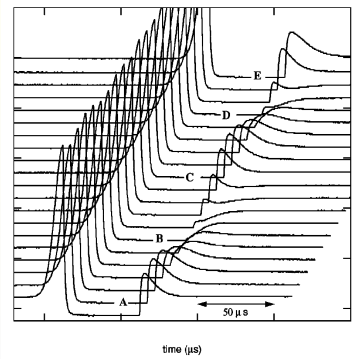

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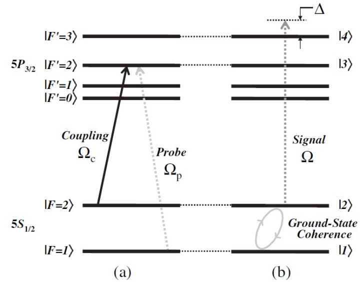
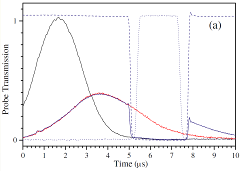
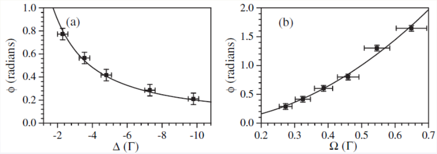
M.D. Lukin *et al*, Phys. Rev. A **65**, 031802(R) (2002).

XPM based on stored light pulses

Manipulate the phase of ground state coherence by storage technique.



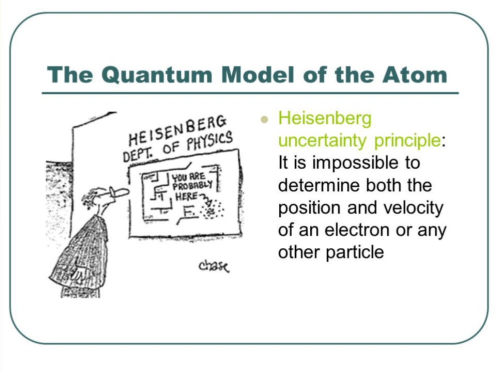


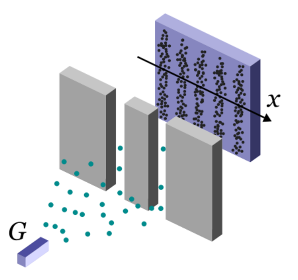


Ite A. Yu *et al*, Phys. Rev. Lett. **96**, 043603 (2006).

Quantum uncertainty

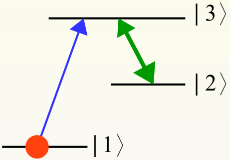
Due to uncertainty principle $\Delta X \Delta P \geq \frac{\hbar}{2}$, we can't measure our physical quantities with 100% precision.





Basic physics of EIT : destructive interference

Quantum interference processes:



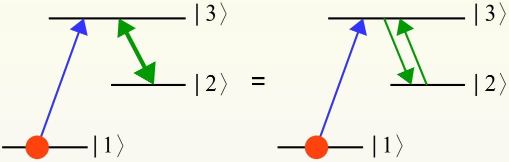
Transition amplitude:

$$A_T = A(1 \to 3) + A(1 \to 3 \to 2 \to 3) + \ldots$$

Transition Probability : $P(1 \to 3) = |A_T|^2 = |\sum_{i \in paths} A_i|^2$

Basic physics of EIT : destructive interference

Quantum interference processes:



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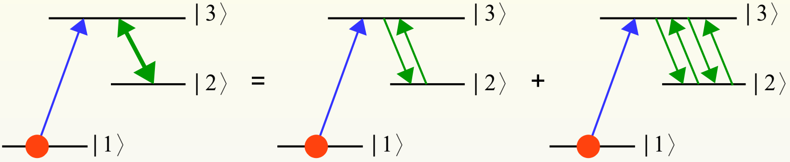

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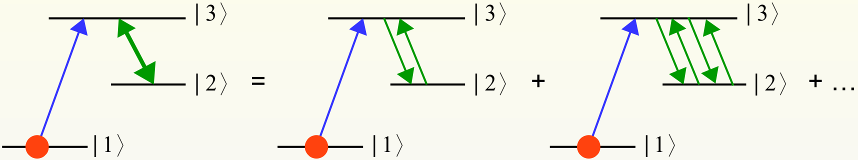

Transition amplitude:

$$A_T = A(1 \rightarrow 3) + A(1 \rightarrow 3 \rightarrow 2 \rightarrow 3) + ...$$

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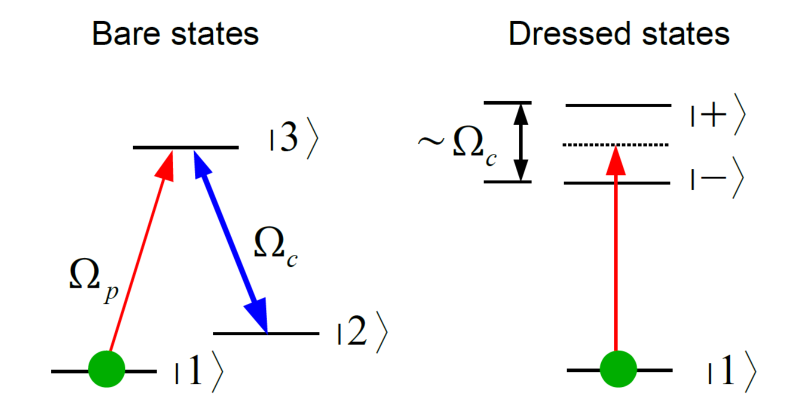
Dressed state picture



$$|D\rangle = \frac{\Omega_c|1\rangle - \Omega_p|2\rangle}{\sqrt{|\Omega_p|^2 + |\Omega_c|^2}} \ , \ \hat{H}_{int}|D\rangle = 0$$

Dark-state polariton

VOLUME 84, NUMBER 22 PHYSICAL REVIEW LETTERS 29 MAY 2000

Dark-State Polaritons in Electromagnetically Induced Transparency

M. Fleischhauer[1] and M. D. Lukin[2]
[1]*Sektion Physik, Ludwig-Maximilians-Universität München, Theresienstrasse 37, D-80333 München, Germany*
[2]*ITAMP, Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138*
(Received 26 January 2000)

We identify form-stable coupled excitations of light and matter ("dark-state polaritons") associated with the propagation of quantum fields in electromagnetically induced transparency. The properties of dark-state polaritons such as the group velocity are determined by the mixing angle between light and matter components and can be controlled by an external coherent field as the pulse propagates. In particular, light pulses can be decelerated and "trapped" in which case their shape and quantum state are mapped onto metastable collective states of matter. Possible applications of this reversible coherent-control technique are discussed.

Dark-state polariton:
superposition of field photon and atomic ground coherence

Dark-state polariton

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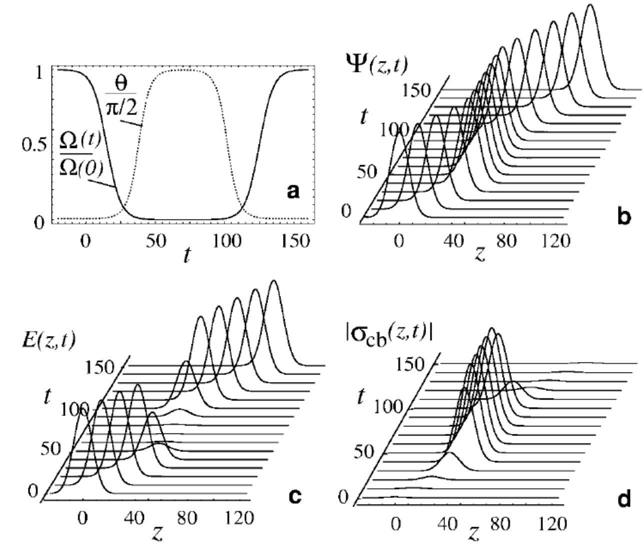
Dark-state polariton:
superposition of field photon and atomic ground coherence

$$\hat{\Psi} = \cos\theta(t)\hat{E} - \sqrt{N}\sin\theta(t)\hat{\sigma}_{12}$$

$$\left[\frac{\partial}{\partial t} + c\cos^2\theta(t)\frac{\partial}{\partial z}\right]\hat{\Psi} = 0$$

Quantum state transfer between photon and atom



M. Fleischhauer and M. D. Lukin, PRL 84, 5094 (2000)

Contents

Motivation : generation of quantum light sources

Squeezed light

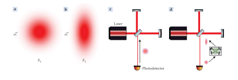
- precision measurement

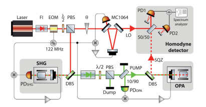
CV entanglement

- deterministic, unconditional
- easily manipulate and observe
- quantum teleportation, quantum dense coding, quantum swapping, quantum communication, etc.

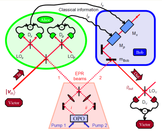
Atom-field interaction

- intrinsic nonlinearity
- controllable

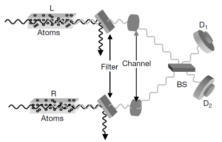




PRL **117**, 110801 (2016)



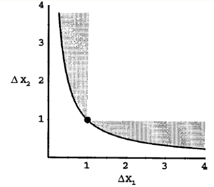
“Unconditional Quantum Teleportation,” Science **282**, 706 (1998).

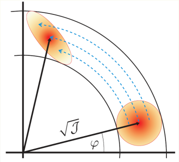


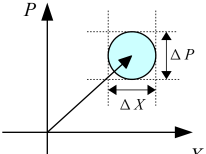
Squeezed light

- below vacuum fluctuation
- non-classical state light
- two-quadrature operator $\left[\hat{X}_1, \hat{X}_2\right] = 2i$
- uncertainty principle: $\Delta X_1 \Delta X_2 \geq 1$
- continuous variable in QIS

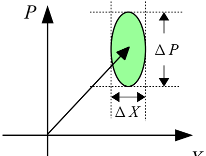
- $\Delta X_1 = e^{-2r}$, $\Delta X_2 = e^{+2r}$
- nonlinearity from self phase modulation (SPM)



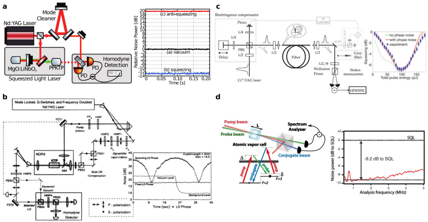




Coherent state

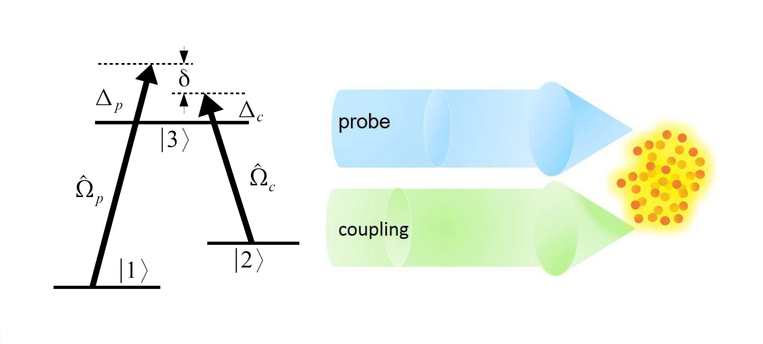


Squeezed state



Phys. Scr. 91 (2016) 053001

Atomic system interacting with light fields



$$\hat{H} = -\hbar\left[\Delta_p\hat{\sigma}_{33}(z,t) + (\Delta_p - \Delta_c)\hat{\sigma}_{22}(z,t)\right] - \hbar\left[\frac{\hat{\Omega}_p(z,t)}{2}\hat{\sigma}_{31}(z,t) + \frac{\hat{\Omega}_c(z,t)}{2}\hat{\sigma}_{32}(z,t) + H.C.\right],$$

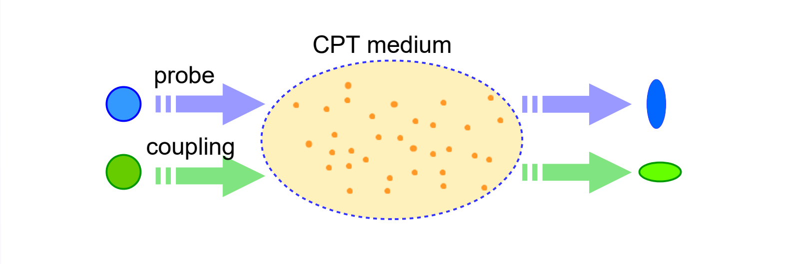
- two-photon detuning $\delta = \Delta_p - \Delta_c$
- asymmetric configuration $\Delta_p = -\Delta_c = \delta/2$

Minimum quadrature variance and correlations

- quadrature operator of field defined by $\hat{X}(\theta) = e^{-i\theta}\hat{a} + e^{i\theta}\hat{a}^\dagger$
- *minimum* quadrature variance :

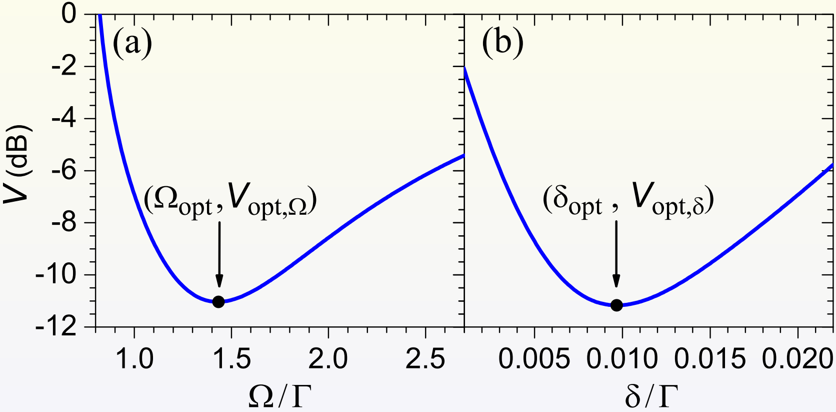
$$V \equiv \langle \Delta\hat{X}^2(\theta_{\rm opt})\rangle = -|\langle\hat{a}^2\rangle| - |\langle\hat{a}^{\dagger 2}\rangle| + 2\langle\hat{a}^\dagger\hat{a}\rangle + 1,$$
while $\theta_{\rm opt} = \left(\mathrm{Arg}[\langle\hat{a}^2\rangle] \pm \pi\right)/2$.

- input coherent states $V_p = V_c = 1$.
- A pair squeezed states are generated at output.



Finding the optimum output variance

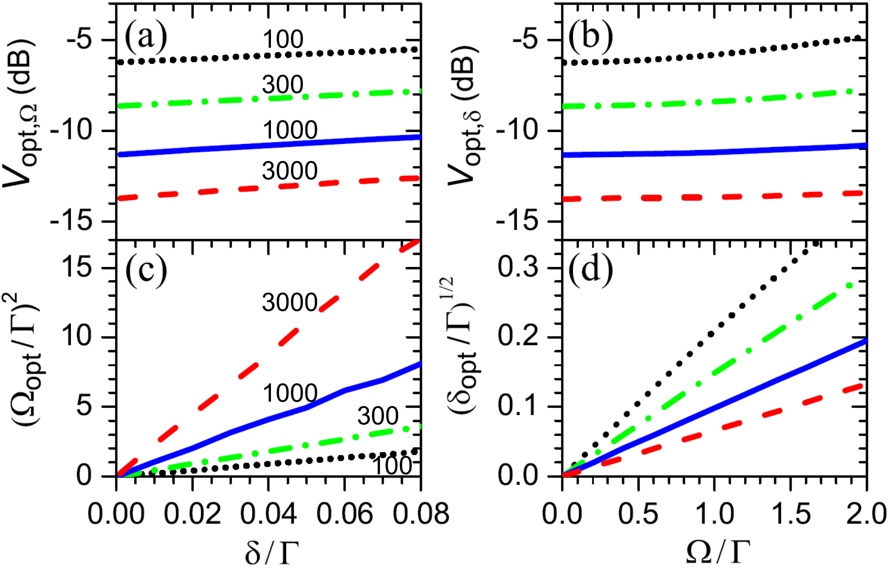
output variance : $V = V(\alpha, \delta, \Omega)$



(a) $\alpha = 1,000$, and $\delta = 0.02\Gamma$

(b) $\alpha = 1,000$, and $\Omega = 1.0\Gamma$

Optimized squeezing



Analytical study for numerical results

$$\frac{\partial}{\partial z}\hat{a}_p = P\hat{a}_p + Q\hat{a}_p^\dagger + R\hat{a}_c + S\hat{a}_c^\dagger + \hat{n}_p$$

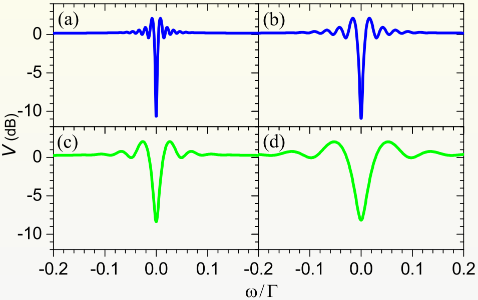
output variance

$$V \approx \left(\sqrt{|Q|^2+1} - |Q|\right)^2 + \frac{Z(2+Z)}{3+2Z}$$

in which $|Q| = \alpha\epsilon/4$, and $Z = \frac{\alpha\epsilon^2}{2}(1+(\Omega/2\Gamma)^2)$. $\epsilon \equiv \Gamma\delta/|\Omega|^2$

- $|Q| \propto$ the propagation delay time of CPT.
- $Z \propto$ the attenuation in CPT.
- variance $\propto (\alpha\epsilon)^{-2}$, noise $\propto \alpha\epsilon^2$.
- $\epsilon_{\text{opt}} \propto \alpha^{-3/4} \Rightarrow V_{opt} \propto 1/\sqrt{\alpha}$
- one-order-of-magnitude increment of OD resulting in 5-dB enhancement of squeezing.

Squeezing spectrum



(a,b) $\alpha = 1000$, (c,d) $\alpha = 300$
$\delta/\Omega^2 \simeq const.$

- time-dependent fluctuation
- The bandwidth of squeezing spectrum $\simeq \Omega^2/\Gamma\sqrt{2\alpha}$.
- oscillation period $T_{osc} = 2\pi \times [2\Omega^2/(\alpha\Gamma)]$.
- The phase change $\phi \approx 2\omega T_D$

Contents

Inseparability criterion for CV systems

- a quantum state of two modes is separable if it can be expressed in the following form:

$$\rho_{12} = \sum_i p_i \rho_{i1} \otimes \rho_{i2}\ ,\ \text{where}\ \sum_i p_i = 1.$$

- entangled continuous variable state - a pair of EPR-type operators, such as $\hat{x}_1 + \hat{x}_2$ and $\hat{p}_1 - \hat{p}_2$.

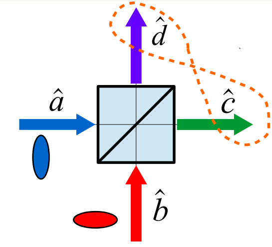
- $\langle[\Delta(\hat{x}_1 + \hat{x}_2)]^2\rangle + \langle[\Delta(\hat{p}_1 - \hat{p}_2)]^2\rangle < 4$
- $\langle[\Delta(\hat{x}_1 + \hat{x}_2)]^2\rangle\langle[\Delta(\hat{p}_1 - \hat{p}_2)]^2\rangle < 1$

L.M. Duan, G. Giedke, J.I. Cirac, and P. Zoller, PRL **84**, 2722 (2000)

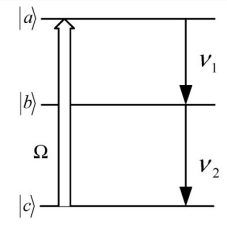
S. Mancini, V. Giovannetti, D. Vitali, and P. Tombesi, PRL **88**, 120401 (2002)

CV entanglement generation

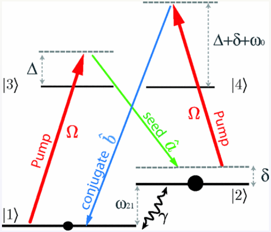
- squeezed light with beam-splitter (BS)
- correlated spontaneous emission laser (CEL)
- four-wave mixing (FWM)
- nondegenrate optical parametric amplification (NDOPA)



PRA **65**, 032323 (2002)



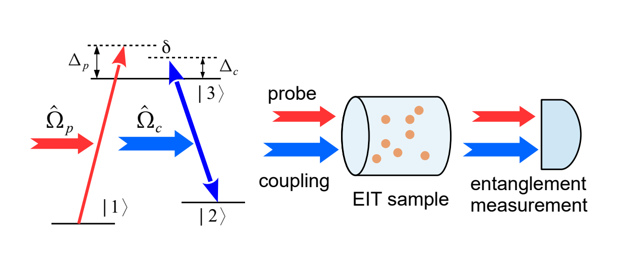
PRL **94**, 023601 (2005)



PRA **82**, 033819 (2010)

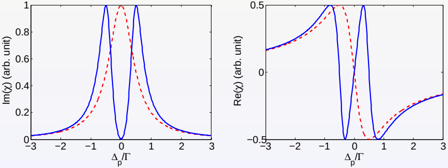
Electromagnetically induced transparency system

CV Entanglement generation from Λ-type EIT system



EIT optical properties

- $\Omega_p \ll \Omega_c$
- destructive interference
- slow light
- light storage and retrieval

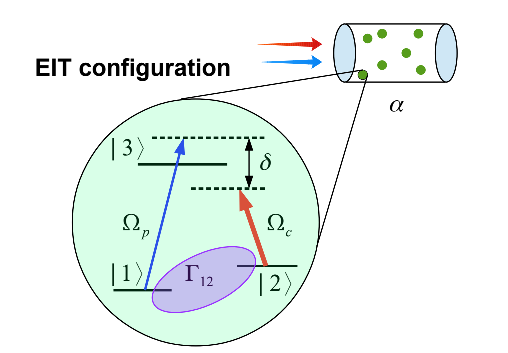


Tunable parameters in EIT system

Entanglement function:

$$V = V(\alpha, \delta, \Omega_p, \Omega_c) = 4\left(1 + \langle \hat{a}_p^\dagger \hat{a}_p \rangle + \langle \hat{a}_c^\dagger \hat{a}_c \rangle - 2|\langle \hat{a}_p \hat{a}_c \rangle|\right)$$

- optical density α
- Rabi frequencies of fields Ω_p and Ω_c
- two-photon detuing δ
- ground state decoherence Γ_{12}



1. Propagation equations of two field fluctuation operators:

$$\frac{\partial}{\partial z}\hat{a}_p = P_1\hat{a}_p + Q_1\hat{a}_p^\dagger + R_1\hat{a}_c + S_1\hat{a}_c^\dagger + \hat{n}_p$$
$$\frac{\partial}{\partial z}\hat{a}_c = P_2\hat{a}_p + Q_2\hat{a}_p^\dagger + R_2\hat{a}_c + S_2\hat{a}_c^\dagger + \hat{n}_c$$

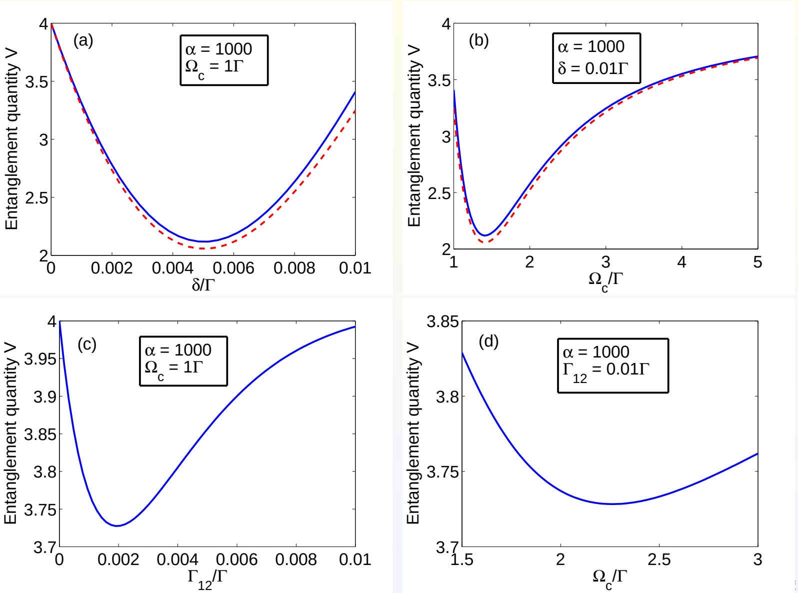
2. commutation relations :
$\left[\hat{a}_p, \hat{a}_p^\dagger\right] = 1 = \left[\hat{a}_c, \hat{a}_c^\dagger\right]$, and $\left[\hat{a}_p, \hat{a}_c\right] = 0 = \left[\hat{a}_p, \hat{a}_c^\dagger\right]$.

3. In EIT region ($\Omega_p \ll \Omega_c$), the coefficients are given by

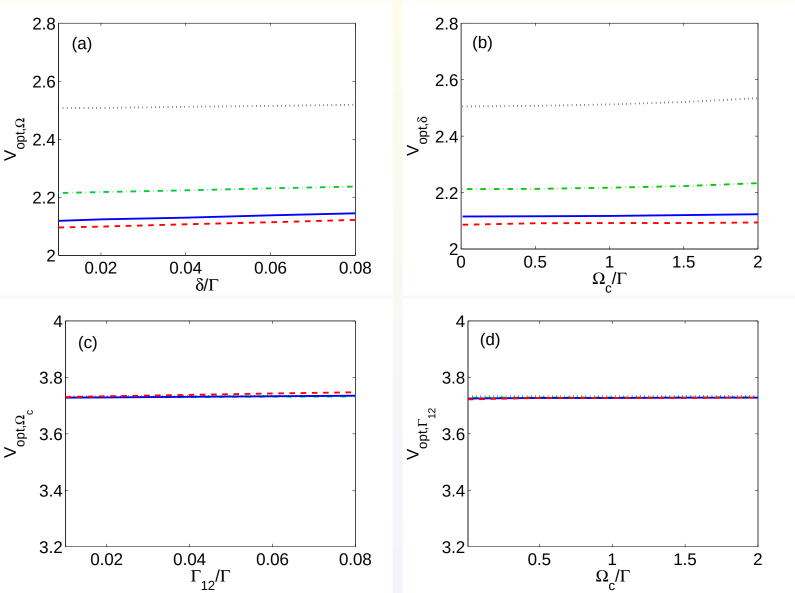
self-damping:	$P_1 \simeq i\alpha\epsilon - 2\alpha\epsilon^2$,	$R_2 \simeq i\alpha\epsilon r^2$
self-squeezing:	$Q_1 \simeq -2i\alpha\epsilon r^2 e^{2iKz}$,	$S_2 \simeq 2i\alpha\epsilon r^2$
cross-damping:	$R_1 \simeq -i\alpha\epsilon r e^{iKz}$,	$P_2 \simeq -i\alpha\epsilon r e^{-iKz}$
cross-correlation:	$S_1 \simeq -i\alpha\epsilon r e^{iKz}$,	$Q_2 \simeq -i\alpha\epsilon r e^{iKz}$

$$K \equiv \alpha\epsilon\ ,\ \ \epsilon \equiv \Gamma\delta/\Omega_c^2\ ,\ \ r \equiv |\Omega_p/\Omega_c| \ll 1.$$

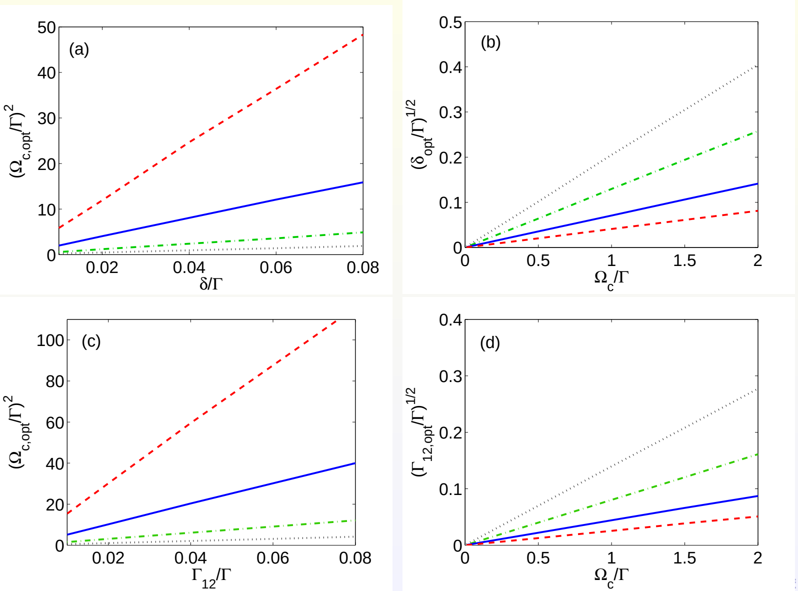
$V(\Omega_c, \delta)$ and $V(\Omega_c, \Gamma_{12})$



Optimum entanglement quantity V_{opt}



Relationship between $\Omega_{c,opt}(\delta_{opt})$ and $\delta(\Omega_c)$



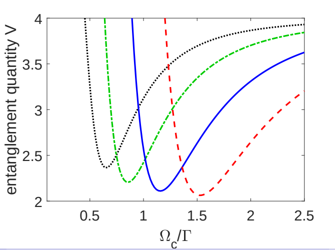
Analytical study

The closed form of output entanglement function is given by

$$V = 4\left(1 + 2\mu^2 - 2\mu\right) + 4\mu\lambda\left(1 - 4\mu/3\right) + 8(2\mu r)^2(1 + \mu^2 - \mu)$$

in which $\mu \equiv \alpha\epsilon r$, and $\lambda \equiv 2\alpha\epsilon^2$ being the damping of probe field.

- When $\lambda = 0$ (no damping) and $r \to 0$, we have $V = 4\left(1 + 2\mu^2 - 2\mu\right)$. $V_{\text{opt}} = 2$, $\forall\alpha$ as long as $\mu = 1/2$.
- $r_{\text{opt}} \propto \alpha^{-1/4}$, $\epsilon_{\text{opt}} \propto \alpha^{-3/4} \Rightarrow (V_{\text{opt}} - 2) \propto \alpha^{-1/2}$



Contents

Conclusion

squeezed light generation from CPT - PRA **96**, 053818 (2017)

1. A pair squeezed lights are generated by CPT media.
2. Optical-density greatly enhances output squeezing without using optical cavities.
3. optimum squeezing $V_{\text{opt}} \propto \alpha^{-1/2}$

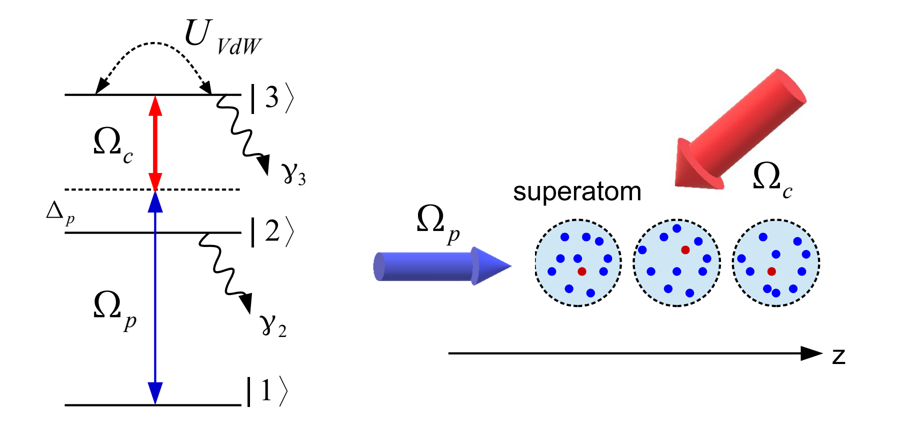
entangled light generation from EIT

1. CV entanglement can be generated by EIT medium.
2. Entanglement generated with two-photon detuning δ is more efficient than that with ground state decoherence rate Γ_{12}.
3. optimum entanglement $(V_{\text{opt}} - 2) \propto \alpha^{-1/2}$

Contents

Rydberg EIT system

- intrinsic nonlinearity
- nonlocality
- many-body effect - Rydberg blockade
- quantum nonlinear optics in low-light level



Thanks for your attention !

生命在祂裡頭，這生命就是人的光。
- 約翰福音1:4

In Him was life, and that life was the light of all mankind.
- John 1:4