Quantum squeezing and entanglement in coherent atomic systems

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Electromagnetically Induced Transparency (EIT)
Electromagnetically Induced Transparency (EIT)

Two-Level Atom

Three-Level Atom

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Electromagnetically Induced Transparency (EIT)

Two-Level Atom

Three-Level Atom

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dispersion and absorption

The susceptibility of EIT medium is

\[ \chi = \frac{N|\mu|^2}{\epsilon_0 \hbar} \frac{i(\gamma - i(\Delta_p - \Delta_c))}{(\Gamma/2 - i\Delta_p)(\gamma - i(\Delta_p - \Delta_c)) + |\Omega_c|^2/4}, \]

group velocity: \[ v_g = c \left[ n + \frac{\omega_p}{2} \frac{\partial \text{Re}(\chi)}{\partial \omega_p} \right]^{-1}. \]
dispersion and absorption

The susceptibility of EIT medium is

\[
\chi = \frac{N|\mu|^2}{\epsilon_0 \hbar} \frac{i (\gamma - i(\Delta_p - \Delta_c))}{(\Gamma/2 - i\Delta_p)(\gamma - i(\Delta_p - \Delta_c)) + |\Omega_c|^2/4},
\]

\[
\text{group velocity} : \quad v_g = c \left[ n + \frac{\omega_p}{2} \frac{\partial \text{Re}(\chi)}{\partial \omega_p} \right]^{-1}.
\]
Electromagnetically induced transparency is a technique for eliminating the effect of a medium on a propagating beam of electromagnetic radiation.

**Optical properties:**
- high transmission
- large refractive index
- enhancement of $\chi^{(3)}$ nonlinearity

**Applications:**
- ultraslow light
- light storage
- quantum memory
- laser without inversion
- precision spectroscopy
Light speed reduction to 17 metres per second in an ultracold atomic gas

Lene Vestergaard Hau*,†, S. E. Harris*, Zachary Dutton*† & Cyrus H. Behroozi*§
Spatial compression of optical pulse in EIT

Storage and retrieval of optical pulse

D. F. Phillips et al., PRL 86, 783 (2001)

You-Lin Chuang (Physics Division, National Center of Theoretical Science, Hsinchu 300, Taiwan)

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Observation of coherent optical information storage in an atomic medium using halted light pulses

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& Lene Vestergaard Hau*†‡

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Quantum squeezing and entanglement in coherent atomic systems

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Coupling intensity (mW cm⁻²)

Normalized probe intensity

Time (μs)

---

Transmission

Probe pulse storage time (μs)

0

0.05

0.1

0.15

0.2

0

250

500

750

1,000

1,250

1,500

0

10

20

30

40

Time (μs)

---

100 Edwin H. Laven

d Applied Sciences, † Department of Physics, Medford, Massachusetts 02138, USA
Beat-note interferometer for direct phase measurement of photonic information

Yong-Fan Chen, Yu-Chen Liu, Zen-Hsiang Tsai, Shih-Hao Wang, and Ite A. Yu

Department of Physics, National Tsing Hua University, Hsinchu, Taiwan

(Received 19 February 2005; published 14 September 2005)

The storage and retrieval of the probe pulse described above is a coherent process [16,17]. This is demonstrated intuitively in Fig. 2(b) with the beat-note interferometer. The black and gray beat notes are the signals from PD1 and PD2, respectively. If the phases of the incoming and outgoing waves of the storage are uncorrelated, there is no beat signal in the retrieved pulse. The phases in different parts of the probe pulse were quantitatively examined, as shown in Figs. 2(c)–2(e). We triggered the oscilloscope by the reference beat note. A rf switch (Mini-Circuits ZFSWA-1-20) was employed to select the Gaussian peak, and its output provided the trigger signal. The oscilloscope waited a certain delay time and then acquired data from the two detectors. We found the instability of a 1-ms delay of the oscilloscope to be equivalent to a phase jitter of $\pm 1.5^\circ$ at the beat frequency of 80 MHz. The driving frequency of the AOM or the beat frequency is sufficiently stable that we are able to extrapolate the reference beat note in Figs. 2(d) and 2(e). Within the measurement accuracy, the phase of the retrieved wave perfectly evolves from the incoming wave and there is no observable phase jump caused by the switching process.
Phase coherence and control

\[ \Phi = (g_+ - g_-) \frac{\mu_B}{\hbar} \int_0^T dt' B(t') \]

Phase coherence and control

\[ \Phi = (g_+ - g_-) \frac{\mu_B}{\hbar} \int_0^T dt' B(t') \]

Phase coherence and control

phase shift: \[ \Phi = (g_+ - g_-) \frac{\mu_B}{\hbar} \int_0^T dt' B(t') \]


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XPM based on stored light pulses

Manipulate the phase of ground state coherence by storage technique.

Quantum uncertainty

Due to uncertainty principle $\Delta X \Delta P \geq \frac{\hbar}{2}$, we can’t measure our physical quantities with 100% precision.

Heisenberg uncertainty principle:
It is impossible to determine both the position and velocity of an electron or any other particle.
Basic physics of EIT: destructive interference

Quantum interference processes:

\[ |1\rangle \rightarrow |3\rangle \]

\[ |1\rangle \rightarrow |2\rangle \rightarrow |3\rangle \]

Transition amplitude:

\[ A_T = A(1 \rightarrow 3) + A(1 \rightarrow 3 \rightarrow 2 \rightarrow 3) + \ldots \]

Transition Probability:

\[ P(1 \rightarrow 3) = |A_T|^2 = \left| \sum_{i \in \text{paths}} A_i \right|^2 \]
Basic physics of EIT: destructive interference

Quantum interference processes:

\[ |1\rangle |2\rangle = |1\rangle |2\rangle \]

Transition amplitude:

\[ A_T = A(1 \rightarrow 3) + A(1 \rightarrow 3 \rightarrow 2 \rightarrow 3) + \ldots \]

Transition Probability: \[ P(1 \rightarrow 3) = |A_T|^2 = \sum_{i \in \text{paths}} |A_i|^2 \]
Basic physics of EIT: destructive interference

Quantum interference processes:

\[ |1\rangle \rightarrow |2\rangle \rightarrow |3\rangle = |1\rangle \rightarrow |2\rangle + |1\rangle \rightarrow |3\rangle + \ldots \]

Transition amplitude:

\[ A_T = A(1 \rightarrow 3) + A(1 \rightarrow 3 \rightarrow 2 \rightarrow 3) + \ldots \]

Transition Probability:

\[ P(1 \rightarrow 3) = |A_T|^2 = \sum_{i \in \text{paths}} |A_i|^2 \]
Basic physics of EIT: destructive interference

Quantum interference processes:

\[
|1\rangle \rightarrow |2\rangle = |2\rangle + |1\rangle + ... \\
|2\rangle \rightarrow |3\rangle = |3\rangle + |2\rangle + ... \\
|1\rangle \rightarrow |2\rangle = |2\rangle + |1\rangle + ... 
\]

Transition amplitude:

\[
A_T = A(1 \rightarrow 3) + A(1 \rightarrow 3 \rightarrow 2 \rightarrow 3) + ...
\]

Transition Probability:

\[
P(1 \rightarrow 3) = |A_T|^2 = \sum_{i \in \text{paths}} |A_i|^2
\]
\[ |D\rangle = \frac{\Omega_c |1\rangle - \Omega_p |2\rangle}{\sqrt{|\Omega_p|^2 + |\Omega_c|^2}}, \quad \hat{H}_{int} |D\rangle = 0 \]
Dark-state polariton

superposition of field photon and atomic ground coherence
Dark-state polariton

superposition of field photon and atomic ground coherence

\[
\hat{\Psi} = \cos \theta(t) \hat{E} - \sqrt{N} \sin \theta(t) \hat{\sigma}_{12}
\]

\[
\left[ \frac{\partial}{\partial t} + c \cos^2 \theta(t) \frac{\partial}{\partial z} \right] \hat{\Psi} = 0
\]
Quantum state transfer between photon and atom

M. Fleischhauer and M. D. Lukin, PRL 84, 5094 (2000)

M. Fleischhauer and M. D. Lukin, PRL 84, 5094 (2000)

You-Lin Chuang (Physics Division, National Center of Theoretical Science, Hsinchu 300, Taiwan)

Quantum squeezing and entanglement in coherent atomic systems

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Motivation: generation of quantum light sources

Squeezed light
- precision measurement

CV entanglement
- deterministic, unconditional
- easily manipulate and observe
- quantum teleportation,
  quantum dense coding,
  quantum swapping,
  quantum communication, etc.

Atom-field interaction
- intrinsic nonlinearity
- controllable

Squeezed light

- below vacuum fluctuation
- non-classical state light
- two-quadrature operator 
  \[ \hat{X}_1, \hat{X}_2 \] = 2i
- uncertainty principle: 
  \[ \Delta X_1 \Delta X_2 \geq 1 \]
- continuous variable in QIS

- \( \Delta X_1 = e^{-2r}, \Delta X_2 = e^{+2r} \)
- nonlinearity from self phase modulation (SPM)
Atomic system interacting with light fields

\[
\hat{H} = -\hbar \left[ \Delta_p \hat{\sigma}_{33}(z, t) + (\Delta_p - \Delta_c) \hat{\sigma}_{22}(z, t) \right] \\
- \hbar \left[ \frac{\hat{\Omega}_p(z, t)}{2} \hat{\sigma}_{31}(z, t) + \frac{\hat{\Omega}_c(z, t)}{2} \hat{\sigma}_{32}(z, t) + H.C. \right],
\]

- two-photon detuning \( \delta = \Delta_p - \Delta_c \)
- asymmetric configuration \( \Delta_p = -\Delta_c = \delta/2 \)
Minimum quadrature variance and correlations

- Quadrature operator of field defined by \( \hat{X}(\theta) = e^{-i\theta} \hat{a} + e^{i\theta} \hat{a}^\dagger \)
- Minimum quadrature variance:
  \[
  V \equiv \langle \Delta \hat{X}^2(\theta_{\text{opt}}) \rangle = -|\langle \hat{a}^2 \rangle| - |\langle \hat{a}^\dagger^2 \rangle| + 2\langle \hat{a}^\dagger \hat{a} \rangle + 1,
  \]
  while \( \theta_{\text{opt}} = \left( \text{Arg}[\langle \hat{a}^2 \rangle] \pm \pi \right) / 2. \)

- Input coherent states \( V_p = V_c = 1. \)
- A pair squeezed states are generated at output.

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Finding the optimum output variance

output variance : \( V = V(\alpha, \delta, \Omega) \)

(a) \( \alpha = 1,000 \), and \( \delta = 0.02\Gamma \)

(b) \( \alpha = 1,000 \), and \( \Omega = 1.0\Gamma \)
Analytical study for numerical results

\[ \frac{\partial}{\partial z} \hat{a}_p = P \hat{a}_p + Q \hat{a}_p^\dagger + R \hat{a}_c + S \hat{a}_c^\dagger + \hat{n}_p \]

**output variance**

\[ V \approx \left( \sqrt{|Q|^2 + 1} - |Q| \right)^2 + \frac{Z(2 + Z)}{3 + 2Z} \]

where \( |Q| = \alpha \epsilon / 4 \), and \( Z = \frac{\alpha \epsilon^2}{2} \left( 1 + (\Omega / 2\Gamma)^2 \right) \). \( \epsilon \equiv \Gamma \delta / |\Omega|^2 \)

- \( |Q| \propto \) the propagation delay time of CPT.
- \( Z \propto \) the attenuation in CPT.
- variance \( \propto (\alpha \epsilon)^{-2} \), noise \( \propto \alpha \epsilon^2 \).
- \( \epsilon_{opt} \propto \alpha^{-3/4} \Rightarrow V_{opt} \propto 1 / \sqrt{\alpha} \)
- one-order-of-magnitude increment of OD resulting in 5-dB enhancement of squeezing.
Squeezing spectrum

\[(a,b) \, \alpha = 1000 \, , \, (c,d) \, \alpha = 300 \]
\[\delta / \Omega^2 \simeq \text{const}.\]

- time-dependent fluctuation
- The bandwidth of squeezing spectrum \( \simeq \Omega^2 / \Gamma \sqrt{2\alpha} \).
- oscillation period \( T_{osc} = 2\pi \times [2\Omega^2 / (\alpha \Gamma)] \).
- The phase change \( \phi \approx 2\omega T_D \).
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Inseparability criterion for CV systems

- a quantum state of two modes is separable if it can be expressed in the following form:

\[ \rho_{12} = \sum_i p_i \rho_{i1} \otimes \rho_{i2} , \text{ where } \sum_i p_i = 1. \]

- entangled continuous variable state - a pair of EPR-type operators, such as \( \hat{x}_1 + \hat{x}_2 \) and \( \hat{p}_1 - \hat{p}_2 \).

\[
\langle [\Delta(\hat{x}_1 + \hat{x}_2)]^2 \rangle + \langle [\Delta(\hat{p}_1 - \hat{p}_2)]^2 \rangle < 4
\]

\[
\langle [\Delta(\hat{x}_1 + \hat{x}_2)]^2 \rangle \langle [\Delta(\hat{p}_1 - \hat{p}_2)]^2 \rangle < 1
\]

---


CV entanglement generation

- squeezed light with beam-splitter (BS)
- correlated spontaneous emission laser (CEL)
- four-wave mixing (FWM)
- nondegenerate optical parametric amplification (NDOPA)

PRA 65, 032323 (2002)  
PRL 94, 023601 (2005)  
PRA 82, 033819 (2010)
Electromagnetically induced transparency system

CV Entanglement generation from Λ-type EIT system

EIT optical properties
- $\Omega_p \ll \Omega_c$
- destructive interference
- slow light
- light storage and retrieval
Tunable parameters in EIT system

Entanglement function:
\[ V = V(\alpha, \delta, \Omega_p, \Omega_c) = 4 \left( 1 + \langle \hat{a}_p^\dagger \hat{a}_p \rangle + \langle \hat{a}_c^\dagger \hat{a}_c \rangle - 2|\langle \hat{a}_p \hat{a}_c \rangle| \right) \]

- optical density \( \alpha \)
- Rabi frequencies of fields \( \Omega_p \) and \( \Omega_c \)
- two-photon detuning \( \delta \)
- ground state decoherence \( \Gamma_{12} \)
Propagation equations of two field fluctuation operators:

\[
\frac{\partial}{\partial z} \hat{a}_p = P_1 \hat{a}_p + Q_1 \hat{a}_p^\dagger + R_1 \hat{a}_c + S_1 \hat{a}_c^\dagger + \hat{n}_p
\]

\[
\frac{\partial}{\partial z} \hat{a}_c = P_2 \hat{a}_p + Q_2 \hat{a}_p^\dagger + R_2 \hat{a}_c + S_2 \hat{a}_c^\dagger + \hat{n}_c
\]

Commutation relations:

\[
[\hat{a}_p, \hat{a}_p^\dagger] = 1 = [\hat{a}_c, \hat{a}_c^\dagger], \text{ and } [\hat{a}_p, \hat{a}_c] = 0 = [\hat{a}_p, \hat{a}_c^\dagger].
\]

In EIT region ($\Omega_p \ll \Omega_c$), the coefficients are given by

**self-damping:** \( P_1 \simeq i\alpha \epsilon - 2\alpha \epsilon^2 \), \( R_2 \simeq i\alpha \epsilon r^2 \)

**self-squeezing:** \( Q_1 \simeq -2i\alpha \epsilon r^2 e^{2iKz} \), \( S_2 \simeq 2i\alpha \epsilon r^2 \)

**cross-damping:** \( R_1 \simeq -i\alpha \epsilon re^{iKz} \), \( P_2 \simeq -i\alpha \epsilon re^{-iKz} \)

**cross-correlation:** \( S_1 \simeq -i\alpha \epsilon re^{iKz} \), \( Q_2 \simeq -i\alpha \epsilon re^{iKz} \)

\[ K \equiv \alpha \epsilon \ , \ \epsilon \equiv \Gamma \delta/\Omega_c^2 \ , \ r \equiv |\Omega_p/\Omega_c| \ll 1. \]
$V(\Omega_c, \delta)$ and $V(\Omega_c, \Gamma_{12})$
Optimum entanglement quantity $V_{opt}$

(a) $\delta / \Gamma$

(b) $\Omega_c / \Gamma$

(c) $\Gamma_{12} / \Gamma$

(d) $\Omega_c / \Gamma$

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Relationship between $\Omega_{c,\text{opt}}(\delta_{\text{opt}})$ and $\delta(\Omega_c)$
Analytical study

The closed form of output entanglement function is given by

\[ V = 4 \left( 1 + 2\mu^2 - 2\mu \right) + 4\mu \lambda \left( 1 - 4\mu/3 \right) + 8(2\mu r)^2(1 + \mu^2 - \mu) \]

in which \( \mu \equiv \alpha \epsilon r \), and \( \lambda \equiv 2\alpha \epsilon^2 \) being the damping of probe field.

- When \( \lambda = 0 \) (no damping) and \( r \to 0 \), we have
  \[ V = 4 \left( 1 + 2\mu^2 - 2\mu \right) \]
  \[ V_{\text{opt}} = 2 \], \( \forall \alpha \) as long as \( \mu = 1/2 \).

- \( r_{\text{opt}} \propto \alpha^{-1/4}, \epsilon_{\text{opt}} \propto \alpha^{-3/4} \Rightarrow (V_{\text{opt}} - 2) \propto \alpha^{-1/2} \)
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Conclusion

**squeezed light generation from CPT - PRA 96, 053818 (2017)**

1. A pair squeezed lights are generated by CPT media.
2. Optical-density greatly enhances output squeezing without using optical cavities.
3. Optimum squeezing $V_{\text{opt}} \propto \alpha^{-1/2}$

**entangled light generation from EIT**

1. CV entanglement can be generated by EIT medium.
2. Entanglement generated with two-photon detuning $\delta$ is more efficient than that with ground state decoherence rate $\Gamma_{12}$.
3. Optimum entanglement $(V_{\text{opt}} - 2) \propto \alpha^{-1/2}$
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Rydberg EIT system

- intrinsic nonlinearity
- nonlocality
- many-body effect - Rydberg blockade
- quantum nonlinear optics in low-light level

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Thanks for your attention!

生命在祂裡頭，這生命就是人的光。
- 約翰福音 1:4

In Him was life, and that life was the light of all mankind.
- John 1:4